LOCK EXCHANGE GRAVITY CURRENTS ON UPSLOPING BEDS

Claudia Adducе[1], Marco Leone[1], Valentina Lombardi[1], Giampiero Sciortino[1], Michele La Rocca[1] & Mario Morganti[1]

[1]Department of Civil Engineering, University of Rome "Roma Tre", Italy, Via Vito Volterra 62, 00146, Rome
E-mail: vlombardi@uniroma3.it

Abstract

This work deals with gravity currents moving on upsloping beds investigated by both experimental and numerical simulations. Laboratory experiments were realized by lock exchange release technique in a Perspex tank of rectangular cross section, divided into two reservoirs by a vertical removable gate, one filled with colored salty water and the other one filled with clear fresh water with lower density. When the gate is removed, the dense fluid collapses developing a gravity current under the surrounding fluid. Different values of the bed's slope θ were tested. Each experiment was acquired by a CCD camera and an image analysis technique, based on the threshold method, was applied to measure the space-time evolution of the current's profile and the time history of the front's position.

Numerical simulations were carried out using a one-dimensional two-layer shallow water model, which accounts for both the oscillation of the free surface and the mixing between the two fluids. Two different relations are used to model the entrainment: a modified form of Ellison & Turner's formula (1959)[5] and a relation suggested by Cenedese & Adduce (2010)[2]. A comparison between numerical and experimental results was performed. Numerical simulations show near the lock an area in which the gravity current's velocity is negative, i.e. the dense fluid is moving downslope. Numerical simulations performed with Ellison & Turner's formula are in good agreement with the experimental results.

Introduction

Gravity currents are caused by a density gradient between two fluids and occur both in natural and in industrial flows. The driving force can be due to a dissolved solute (i.e. salt in the sea), to a difference of temperature, or to the presence of suspended sediments. Examples of gravity currents are given by avalanches, turbidity currents, pyroclastic flows, lava flows, sea-breeze and salt wedge propagation (Simpson, 1997[13]).

Many studies investigated gravity currents by both laboratory experiments and numerical simulations and most of the models used in the literature are based on the shallow-water theory (Rottman & Simpson 1983[11]; Shin et al. 2004[12]; La Rocca et al. 2008[10]; Adduce et al. 2010[2]). Rottman & Simpson (1983)[11] studied gravity currents by laboratory experiments and compared measurements with numerical solutions of the shallow water equations for a two-layer fluid bounded at top and bottom by rigid horizontal planes and at one end by a vertical wall, neglecting mixing effects between the two fluids. Benjamin (1968)[1] developed a theory for the propagation of a steadily advancing current and focused the attention on the importance of dissipation in gravity current dynamics. Shin et al. (2004)[12] provided a theory based on the energy-conserving flow that is in agreement with their experiments, and showed that dissipation is not important at high Reynolds number. La Rocca et al. (2008)[10] studied the dynamics of three-dimensional gravity currents on smooth and rough beds by lock exchange experiments and numerical simulations, using a shallow water model considering two layers of immiscible liquids. Adduce et al. (2010)[2] performed lock exchange experiments on a flat bed and compared experimental results with numerical simulations obtained by a two-layer, shallow water model for miscible fluids.

The aim of this paper is the investigation of gravity currents moving on upsloping beds by both laboratory experiments and numerical simulations. Four different bed's slopes were investigated.

Experimental gravity currents were realized in a Perspex channel of a rectangular cross-section, divided in two portions by a vertical sliding gate, as shown in Figure 1. The lock was filled with the heavier fluid, realized by a solution of tap water and salt, while the other volume was filled with the lighter fluid, i.e. fresh water. The experiment starts when the gate is removed, the salty water flows under the lighter fluid and the gravity current develops. The experiment stops when the gravity current reaches the right end wall of the channel. Such experimental technique is called "lock exchange release".

Numerical simulations were performed by 1D, two-layer, shallow water model. The mathematical model takes into account both the oscillation of the free surface and the mixing (i.e. entrainment) between the two fluids.

Entrainment was modeled following two different relations and numerical results obtained were compared with experimental results. The first relation used is the one suggested by Cenedese & Adduce (2010) [2] and the second one is a modified form of Ellison & Turner's (1959)[5] formula. Numerical simulations performed by using Ellison & Turner's formula are in good agreement with the experimental results, suggesting that the shallow water model is a valid instrument to reproduce gravity currents moving on beds with different slopes.

Experimental apparatus

The experiments were conducted at the Hydraulics Laboratory of the University of Rome "Roma Tre", in a Perspex tank of rectangular cross-section, 3.0 m long, 0.3 m deep and 0.2 wide. The tank was divided in two parts by a vertical sliding gate placed at the distance x_0 from the beginning of the channel, as shown in Figure 1. The left part of the tank was filled with salty water with initial density $\rho_{01}>\rho_2$, while the right part was filled with tap water. The depth of the two fluids was h_0. Density measurements were performed by a pycnometer and a small quantity of dye was dissolved into the salty water to allow the visualization of the gravity current during the experiments.

Each experiment was recorded by a CCD camera, with a frequency of 25 Hz, and an image analysis technique, based on a threshold method, was applied to measure the space-time evolution of the gravity current's profile. The conversion factor pixel/cm was obtained using a rule placed along both the horizontal and vertical walls of the channel.

Four experiments were performed keeping constant $\rho_1 \cong 1060$ kg/m^3, ρ_2=1000 kg/m^3, h_0=0.15 m, x_0=0.1 m and varying the bed's sloping angle θ. The values of bed's sloping angles investigated were: +0.00°, -1.14°, -1.39° and -1.52°. θ= -1.39° (i.e. Run 3) is the critical bed's sloping angle for $\rho_1 \cong 1060$ kg/m^3. In this work the critical value of the bed's sloping angle is defined as the angle for which the gravity current reaches the end of the tank with a front's speed close to zero. For Run 2 the gravity current reaches the end wall with a front's speed higher than zero (i.e. subcritical slope), while for Run 4 the current doesn't reach at all the end of the tank (i.e. supercritical slope). Table 1 shows the bed's angles for each experiment.

Table 1: Experimental parameters for all runs

RUN	ρ_{01} [kg/m^3]	θ [°]
1	1059.56	+0.00
2	1059.72	-1.14
3	1059.75	-1.39
4	1059.72	-1.52

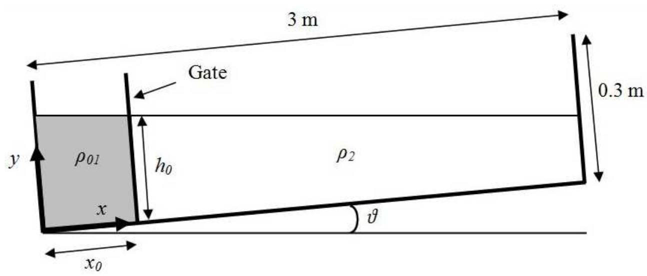

Figure 1: Sketch of the tank used to perform 2D lock release gravity currents.

Mathematical model

A two-layer, 1D, shallow-water model was used to simulate gravity currents. Gravity currents frequently develop along the longitudinal direction, so that the ratio between the depth and the length of the current is small enough to allow the application of the shallow water theory. Several authors investigated gravity currents by shallow water equations (Rottman & Simspon (1983)[11], Sparks et al. (1993)[14], Hogg et al. (1999)[8]).

These authors assumed a steady free surface, while in the present work this hypothesis has been removed in order to have a more realistic solution, modeling the space-time evolution of the free surface. The mathematical model takes also into account the mixing between the two fluids. The entrainment at the interface, due to a mass transport from the lighter fluid to the heavier one, causes a decrease of the density of the gravity current. The entrainment between the two fluids was modeled by both a modified Ellison & Turner's formula (1959)[5] and the Cenedese & Adduce's formula (2010)[2]. Figure 2 shows the frame of reference used in the model.

A monodimensional gravity current moving on a bed of a slope θ is considered. For the mathematical model, negative values of θ are referred to upsloping beds. The heavier current of height h_1 and density ρ_1 flows below the lighter one of height h_2 and density ρ_2. Applying both principle of mass conservation and projecting along the axis x the balance of momentum equations, the following system of hyperbolic partial differential equations is obtained:

$$\begin{cases} \dfrac{\partial(\rho_1 h_1)}{\partial t} + \dfrac{\partial(\rho_1 V_1 h_1)}{\partial x} = \rho_2 V_e \\ \dfrac{\partial(\rho_2 h_2)}{\partial t} + \dfrac{\partial(\rho_2 V_2 h_2)}{\partial x} = -\rho_2 V_e \\ \dfrac{\partial V_1}{\partial t} = g\sin\theta - \dfrac{\partial}{\partial x}\left(\dfrac{\rho_1 h_1 + \rho_2 h_2}{\rho_1} g\cos\theta + \dfrac{V_1^2}{2}\right) - \dfrac{\tau_{1b} - \tau_{12}}{\rho_1 h_1} \\ \dfrac{\partial V_2}{\partial t} = g\sin\theta - \dfrac{\partial}{\partial x}\left[(h_1 + h_2) g\cos\theta + \dfrac{V_2^2}{2}\right] - \dfrac{\tau_{12} + \tau_{2b}}{\rho_2 h_2} \end{cases} \quad (1)$$

where the unknown quantities h_1, h_2, V_1 and V_2 are the depth and the velocity of the lower and the upper layer, respectively, V_e is the entrainment velocity, τ_{1b} and τ_{2b} are the stress terms between the two fluids and the bottom (these terms include both bed's stress and lateral walls stress), and τ_{12} is the stress at the interface between the two fluids. The bottom stress is modeled by Darcy-Weisbach's formula (1858[4], 1845[15]), like in La Rocca et al. (2008)[10]:

$$\begin{cases} \tau_{1b} = \lambda_1 \rho_1 \dfrac{V_1|V_1|}{8} \dfrac{2h_1 + B}{B} \\ \tau_{2b} = \lambda_2 \rho_2 \dfrac{V_2|V_2|}{8} \dfrac{2h_2}{B} \end{cases} \quad (2)$$

where B is cross section's width of tank; λ_i, the friction factor, was defined by Colebrook (1939)[3] for the transition between laminar and turbulent flow:

$$\lambda_i = \lambda_{i\infty}\left(1 + \frac{4h_i}{\mathrm{Re}_i\,\varepsilon}\right)^2 \cong \lambda_{i\infty}\left(1 + \frac{8h_i}{\mathrm{Re}_i\,\varepsilon}\right) \quad (3)$$

where $\lambda_{i\infty}$, Re_i and ε/h_i are the friction factor for turbulent rough flows, the Reynolds number and the relative roughness of the i_{th} layer, respectively. $\lambda_{i\infty}$ corresponds to turbulent flow; the latter parameter and Re_i are defined as:

$$\lambda_{i\infty} = \frac{1}{4}\left[\log\left(\frac{3.71 h_i}{\mathrm{Re}_i\,\varepsilon}\right)\right]^{-2} \quad (4)$$

$$\mathrm{Re}_i = \frac{|V_i| h_i}{\nu_i} \quad (5)$$

Equation (3) shows that the term $8h_i/(\mathrm{Re}_i\,\varepsilon)$ adapts the friction factor for turbulent rough flows to turbulent transition flows. In the performed experiments turbulent transition flows develop.

The stress at the interface between the two fluids is defined as:

$$\tau_{12} = \lambda_{12} \frac{\rho_1 + \rho_2}{2} \frac{(V_2 - V_1)|V_2 - V_1|}{8} \quad (6)$$

where λ_{12} is the friction factor at the interface between two different fluids. The value λ_{12} = 0.24 was found as the optimum value and it was used in previous works. In this study this parameter is expressed as a function of the gravity current's Reynolds number given by (5):

$$\lambda_{12} = \lambda' + \frac{\lambda'' - \lambda'}{1 + e^{-(\mathrm{Re}_1 - \mathrm{Re}_0)}} \quad (7)$$

where $\lambda'' < \lambda'$ and Re_0 is a particular value of Reynolds number; all these parameters were calibrated and the following values were obtained: λ'= 0.24, λ''= 0.19, Re_0= =6000.

In order to model V_e, many authors studied mixing effect at the interface (Hacker et al. (1996)[6], Holford & Linden (1999)[9], Hallworth et al. (1996)[7]). In this study, initially, a modified Ellison & Turner's formula (1959)[5] was used to model the entrainment. V_e is given as a function of Froude number of the gravity current, defined as:

$$Fr_1 = \frac{|V_1|}{\sqrt{h_1 \dfrac{\rho_1 - \rho_2}{\rho_1} g\cos\theta}} \quad (8)$$

Because Ellison & Turner's formula was obtained by an experimental apparatus different from the lock exchange experiment, in this paper some modifications to Ellison & Turner's relation were adopted. So the relation used to model the entrainment parameter is:

$$E = \frac{V_e}{|V_1 - V_2|} = \frac{k \cdot Fr_1^2}{Fr_1^2 + 5} \quad (9)$$

where k is a dimensionless coefficient. The entrainment velocity increases as k increases. The calibration value k=0.95 supplies a correct evaluation of the gravity current's depth and a good simulation of the front's speed of the gravity current.

The results provided by the latter entrainment modeling were compared to the ones derived from the Cenedese & Adduce entrainment formulation. These authors express V_e as a function of both Froude and Reynolds number of the gravity current. So the relation to model the entrainment parameter is:

$$E = \frac{V_e}{|V_1 - V_2|} = \frac{Min + A \cdot Fr_1^{\alpha}}{1 + A \cdot C_{\inf}(Fr_1 + Fr_0)^{\alpha}} \quad (10)$$

where:

$$C_{\inf} = \frac{1}{Max} + \frac{B}{\mathrm{Re}_1^{\beta}} \quad (11)$$

The dimensionless coefficients derived from the calibration carried out by Cenedese & Adduce have the following values: Min=$4\cdot10^{-5}$, A=$3.4\cdot10^{-3}$, Fr_0=0.51, α=7.18, Max=1, B=243.52 and β=0.5. Such parameters are based on experimental data and oceanic measurements from

Mediterranean Sea, Denmark Strait, Faroe Islands, Baltic Sea and Lake Ogawara.

The mathematical model was numerically solved by an explicit Mac-Cormack's finite difference scheme by predictor-corrector scheme. By this way a greater scheme's stability is assured by using modest computing resources.

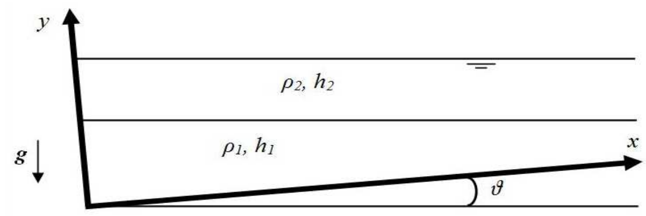

Figure 2: Frame of reference used in the mathematical model.

Results and discussion

Figure 3a-d shows a comparison between experimental front's position and numerical prediction for all the runs: CA indicates the numerical simulations performed with Equation (10) suggested by Cenedese & Adduce (2010)[2] to model the entrainment; ETm stands for the simulations carried out by using the modified Ellison & Turner's formula given by Equation (9) (i.e. k=0.95); ETzero represents the simulation obtained neglecting the entrainment phenomena (i.e. k=0.0). The space scale is the gate position x_0, while the time scale t_0 is defined as:

$$t_0 = \frac{x_0}{\sqrt{g_0' h_0}} \tag{12}$$

From Figure 3a-d a good agreement between experimental data and the numerical front's position predicted using Ellison & Turner's formula (i.e. ETm) to model the entrainment can be observed. Regarding the runs performed on upsloping beds, the simulation performed without taking into account the entrainment term (ETzero) agrees with laboratory data only for the first stage of gravity current's development. The curve resulting from the run performed with the formula by Cenedese & Adduce 2010[2] (i.e. CA) is substantially overlapped to the ETzero curve. In fact CA provides low values of entrainment coefficient (i.e. $E \cong Min$) for the range of Froude numbers investigated in the regime of interest (i.e. $Fr<1$).

In order to define the ability of the model in simulating gravity currents, an error *MPE* (Mean Percentage Error) was computed in the following way:

$$MPE = \frac{100}{N} \sum_{i=1}^{N} \left(\frac{\left| x_{nf,i} - x_{ef,i} \right|}{x_{nf,i}} \right) \tag{13}$$

where x_{nf} and x_{ef} are the numerical and experimental front position, respectively. Table 2 shows the value of *MPE* for each run. As can be observed from Table 2 the best agreement with experimental data is obtained with ETm simulations: the mean error *MPE* reaches a maximum value of 4.30 % for Run 1 and a minimum value of 2.91 % for Run 2. Therefore the agreement between the results for the numerical and experimental front position is fairly good, being the error values reasonable for all the investigated slope's angles.

Table 2: Mean Percentage Error (*MPE*) for each run computed on the basis of Equation (13).

RUN	*MPE* [%]		
	ETm	ETzero	CA
1	4.30	6.53	6.43
2	2.91	27.80	27.34
3	3.21	37.34	36.83
4	3.48	37.35	36.84

Figure 4a-d show lower layer velocity V_1 along x-axis at four different time steps after release for Run 2 performed on upsloping bed. The velocity values resulting from the simulation performed with the formula by Cenedese & Adduce 2010[2] (i.e. CA) are substantially overlapped to the ones predicted neglecting the entrainment term (i.e. ETzero). Although in Figure 4a-b the three different simulations show a similar general trend of V_1, since the third considered time step (Figure 4c-d), V_1 predicted by ETm simulation diverge from both the CA and ETzero simulations. As observed for figure 3a-d, neglecting the entrainment term affects numerical results only after the first stage of the gravity current's development. Furthermore, numerical simulation performed by using the ETm formulation, shows near the lock an area in which the gravity current's velocity is negative, i.e. the dense fluid is moving downslope.

Figure 5 and Figure 6 show the comparison between images acquired by the camera and numerical profiles obtained with CA, Etm and ETzero simulations at four different time steps after release for Run 1 and Run 2, respectively. As explained in the previous section, the effect of mixing is to produce a mass flow from the lighter fluid to the heavier one, causing an increase of the height of the current's profile and therefore a decrease of both the density and the velocity of the gravity current. Therefore, as can be observed in Figure 5 and Figure 6, both the numerical simulation obtained neglecting the entrainment term (i.e. ETzero) and the one performed by using Cenedese & Adduce's formula (i.e. CA) provide a less high profile for the current.

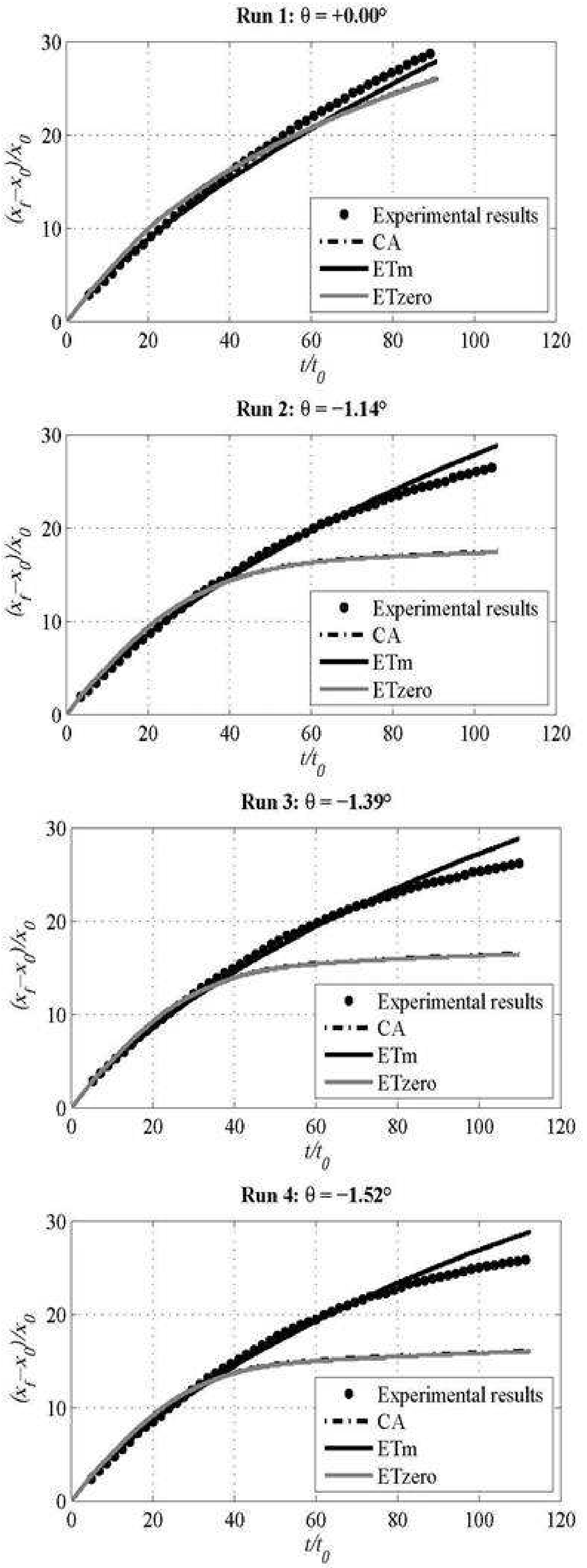

Figure 3a-d: Dimensionless plot of front's position versus time for all the performed runs: experimental data (circle), CA (dash-dot line), ETm (black line), ETzero (grey line).

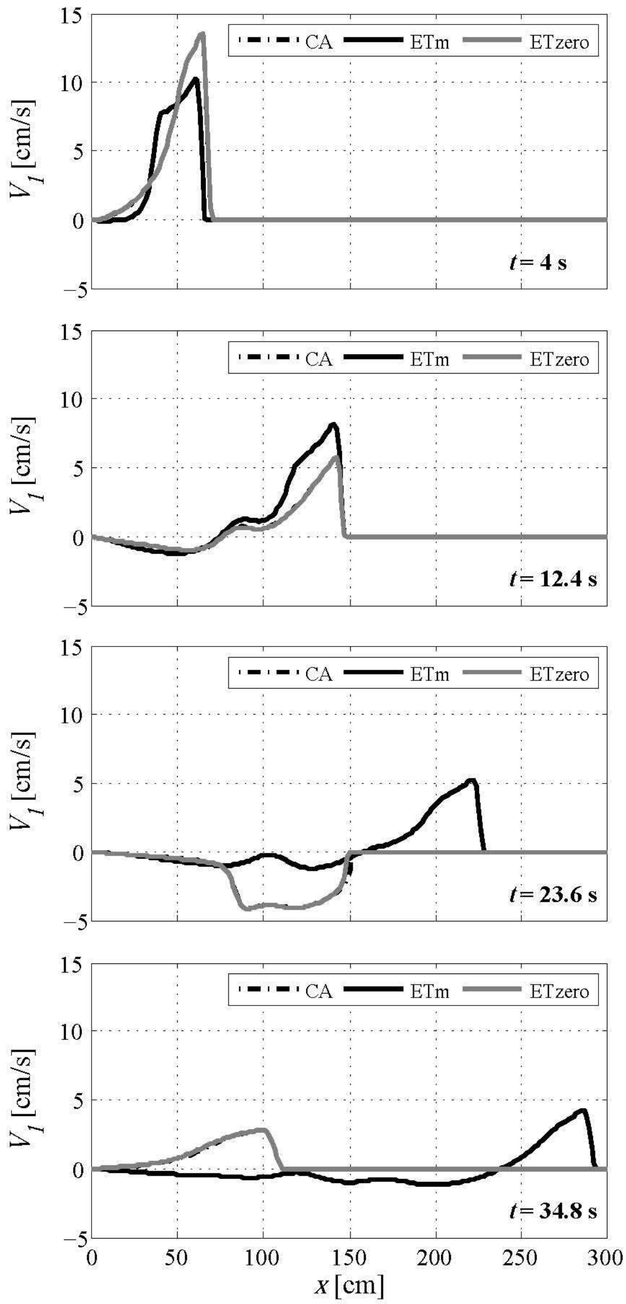

Figure 4a-d: Plot of lower layer velocity V_1 along x-axis at four different time steps for Run 2.

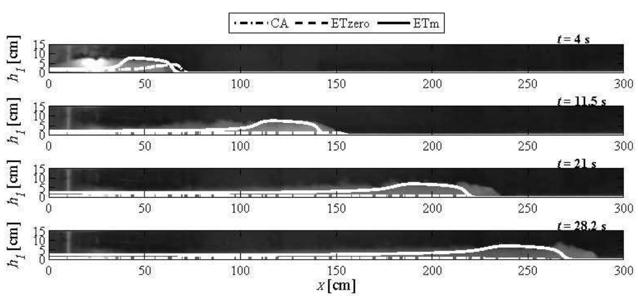

Figure 5: Comparison between numerical simulations and images acquired by the camera at three different time steps after release for Run 1.

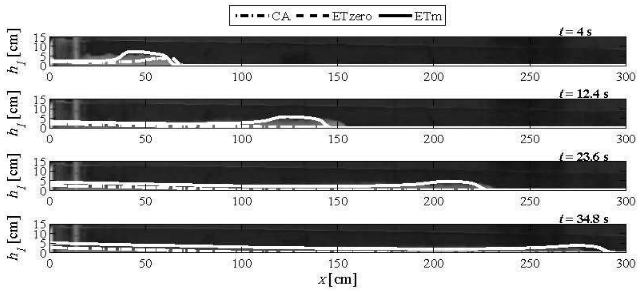

Figure 6: Comparison between numerical simulations and images acquired by the camera at three different time steps after release for Run 2.

Conclusions

Four full depth lock exchange release experiments were performed to study gravity current's dynamics: three of them on upsloping bed. *Particle Image Velocimetry* (PIV) was applied to measure the velocity field of the gravity currents. The main purpose is to test a mathematical model that uses a new formula, in order to take in account also the entrainment phenomenon.

A good agreement between experimental data and the numerical front's position predicted using Ellison & Turner's formula (i.e. ETm) to model the entrainment can be observed. Regarding the runs performed on upsloping beds, the simulation performed without taking into account the entrainment term (ETzero and, de facto, CA) agrees with laboratory data only for the first stage of gravity current's development.

The velocity values resulting from the simulation performed with the formula CA are substantially overlapped to the ones predicted neglecting the entrainment term (i.e. ETzero). Numerical simulation performed by using the ETm formulation, shows near the lock an area in which the gravity current's velocity is negative; moreover, velocity profiles show that the maximum velocity occurs about 5 cm behind the nose of the current.

The numerical simulation obtained neglecting the entrainment term (i.e. ETzero) and the one performed by using Cenedese & Adduce's formula (i.e. CA) provide a less high profile for the current, than using Ellison & Turner's formula (i.e. ETm).

References

[1] Benjamin, T. (1968). Gravity currents and related phenomena. *J. Fluid Mech.*, 209-248.

[2] Cenedese, C., & Adduce, C. (2010). A new entrainment parameterization for mixing in overflows. *Journal of Physical Oceanography*, 1835-1850.

[3] Colebrook, C. (1939). Turbulent flow in pipes, with particular reference to the transition region between the smooth and rough pipe laws. *J. Inst. Civ. Eng.*, 133-156.

[4] Darcy, H. (1858). *Récherches rélatives au mouvement de l'eau dans le tuyaux.* Parigi.

[5] Ellison, T., & Turner, J. (1959). Turbulent entrainment in stratified flows. *J. Fluid Mech.*, 423-448.

[6] Hacker, J., Linden, P., & Dalziel, S. (1996). Mixing in lock-release gravity currents. *Dyn. of Atm. and Oceans*, 183-195.

[7] Hallworth, M., Huppert, H., Phillips, J., & Sparks, R. (1996). Entrainment into two-dimensional and axisymmetric turbulent gravity currents. *J. Fluid Mech.*, 289-311.

[8] Hogg, A., Huppert, H., & Hallworth, M. (1999). Reversing buoyancy of particle-driven gravity currents. *Physics of fluids*, 2891-2900.

[9] Holford, J., & Linden, P. (1999). Turbulent mixing in a stratified fluid. *Dyn. of Atm. and Oceans*, 173-198.

[10] Rocca, M. L., Bateman, A., Sciortino, G., & Adduce, C. (2008). Experimental and numerical simulation of three dimensional gravity currents on smooth and rough bottom.

[11] Rottman, J., & Simpson, J. (1983). Gravity currents produced by instantaneous releases of a heavy fluid in a rectangular channel. *J. Fluid Mech.*, 95-110.

[12] Shin, J., Dalziel, S., & Linden, P. (2004). Gravity currents produced by lock exchange. *J. Fluid Mech.*, 1-34.

[13] Simpson, J. (1997). *Gravity currents: in the Environment and the Laboratory (2nd edition).* Cambridge University Press.

[14] Sparks, R., Bonnecaze, R., Huppert, H., Lister, J., Hallworth, M., Mader, H., & Phillips, J. (1993). Sediment-laden gravity currents with reversing buoyancy. *Earth and Plan. Sc. Let.*, 243-257.

[15] Weisbach, J. (1845). *Lehrbuch der Ingenieur und Masch. – Mechanik I.* Braunschweig.