Soil Dynamics and Earthquake Engineering 55 (2013) 33–43

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

A two-rigid block model for sliding gravity retaining walls

Riccardo Conti [a,*], Giulia M.B. Viggiani [b], Simone Cavallo [c]

[a] SISSA – International School for Advanced Studies, Via Bonomea 265, 34136 Trieste, Italy
[b] Dip. Ingegneria Civile, Università di Roma Tor Vergata, Italy
[c] Università di Roma Tor Vergata, Italy

ARTICLE INFO

Article history:
Received 4 March 2013
Received in revised form
1 August 2013
Accepted 24 August 2013

Keywords:
Gravity retaining walls
Permanent displacements
Earthquake
Sliding block
Active earth pressure

ABSTRACT

This paper presents a new two-rigid block model for sliding gravity retaining walls. Some conceptual limitations of a direct application of Newmark's sliding block method to the case of retaining walls are discussed with reference to a simple scheme of two interacting rigid blocks on an inclined plane. In particular, it is shown that both the internal force between the blocks and their absolute acceleration are not constant during sliding, and must be computed by direct consideration of the dynamic equilibrium and the kinematic constraints for the whole system. The same concepts are extended to the analysis of the active soil wedge–wall system, leading to an extremely simple procedure for computing the relative displacements of the wall when subjected to base accelerations exceeding the critical value. A comparison with the results of numerical analyses demonstrates that the proposed method is capable of describing fully the kinematics of the soil wedge–wall system under dynamic loading. On the contrary, direct application of Newmark's method may lead to inaccurate predictions of the final displacements, in excess or in defect depending on a coefficient which emerges from direct consideration of the dynamic equilibrium of the whole system. This coefficient can be viewed as a corrective factor for the horizontal relative acceleration of the wall, related to the mechanical and geometrical properties of the soil–wall system.

1. Introduction

In recent literature several cases are reported of damage to gravity and cantilevered retaining walls due to severe seismic shaking [9,18]. Well after the seminal works by Okabe [27] and Mononobe and Matsuo [25], several studies have tackled the problem of computing active dynamic pressures with a theoretical [20,24,29,17], experimental [2] and numerical approach [10,8].

More recently, following Newmark [26] and the pioneering work by Richards and Elms [28], the emphasis has gradually shifted towards the computation of wall displacements, in light of performance based design. For instance, Zeng and Steedman [34], Ling [21], Kim et al. [16], Huang [13], Basha and Babu [3], Huang et al. [14] and Trandafir et al. [31] have considered both a translational and rotational failure mechanism for the wall to compute relative displacements using Newmark's approach. The idea is that of defining a critical value of the acceleration, a_c ($=k_c g$), for an active type of soil–wall failure mechanism, and then computing the displacements of the wall due to a base acceleration a ($=k_h g$), by direct double integration of the relative acceleration ($a-a_c$). The underlying assumption is that, when the wall moves relative to its base, the absolute acceleration of the wall remains constant and equal to its critical value [4]. Only a few authors [33,23,31] apply a corrective factor to the relative acceleration, emerging from direct consideration of the equation of motion of the soil–wall system. More in detail, Zarrabi-Kashani [33] proposes an equation in which k_c varies during shaking and an iterative procedure must be adopted to integrate the equation of motion in time; Trandafir et al. [31] use the Bishop simplified slice method to determine k_c, paying less attention to the kinematic conditions at the soil–wall interface when sliding occurs; Michalowski [23] considers a multi-block mechanism derived from slope stability methods yielding an equation of motion that the same author [12] does not recommend for the analysis of retaining walls, due to the formation of multiple shear bands. As a result, wall displacements are still commonly simply obtained by direct double integration of (k_h-k_c)g.

This work examines the dynamic behaviour of gravity retaining walls with dry cohesionless backfill, accumulating relative displacements solely by sliding on their base. A simple scheme of two frictional rigid blocks resting on an inclined plane is examined first to show that Newmark's approach cannot be used directly to compute the relative displacements of the blocks. More in detail, by considering the dynamic equilibrium of the blocks, it is shown that the assumption that the absolute acceleration during sliding is constant and equal to its critical value is not correct, and this affects the internal force between the two blocks.

0267-7261/$ - see front matter
http://dx.doi.org/10.1016/j.soildyn.2013.08.007

Turning the attention to gravity retaining walls, this implies that it is not possible to examine the dynamic equilibrium of the wall on its own, as the dynamic thrust between the active wedge and the wall is an internal force that cannot be computed without knowledge of the absolute acceleration of the soil wedge. The equations of motion of both the soil wedge and the wall are then derived based on dynamic equilibrium and kinematic conditions of the whole system; the predictions of the model are discussed and compared with the results of numerical analyses.

Once again, Newmark's approach cannot be used directly to evaluate the permanent displacements of the soil–wall system unless a corrective factor for the relative accelerations is introduced in the computation. This factor is independent of the seismic excitation and, for realistic geometries of the wall and mechanical properties, takes values between about 0.7 and 1.1.

In the derivation of the model, the vertical component of the base acceleration is neglected ($k_v=0$), as this has been proven to have a minor effect on the seismic displacements of gravity retaining walls (see *e.g.* [33,13,11]). Also, the work does not examine the effects of shear strength reductions due to strain softening of dense sand on shearing.

2. A simple two-rigid blocks model on an inclined plane

Fig. 1 shows the problem under examination, consisting of two frictional rigid blocks resting on a plane with an inclination α on the horizontal. The reference axes, x and y, are parallel and orthogonal to the inclined plane. $W_{1,2}$, $\phi_{1,2}$, and k_h are the weights of the two blocks, the friction angles at the interface between the blocks and the plane, and the coefficient of horizontal base acceleration. Moreover, S is the compressive internal force between the two blocks, which are assumed to have perfect rigid smooth contact, and $T_{1,2}$ and $N_{1,2}$ are the shear and normal forces at the contact between the blocks and the plane.

The dynamic equilibrium of each block, in the x and y directions, is given by

$$\Sigma F_x=\frac{W}{g}\ddot{u} \tag{1a}$$

$$\Sigma F_y=\frac{W}{g}\ddot{v} \tag{1b}$$

in which ΣF_x and ΣF_y are the sums of all the forces acting on the block, while $\ddot{u}$ and $\ddot{v}$ are the absolute accelerations of the block, in the x and y directions, respectively.

$$\ddot{u}=\ddot{u}_{base}+\ddot{u}_r=k_h g\ \cos\alpha+\ddot{u}_r \tag{2a}$$

$$\ddot{v}=\ddot{v}_{base}+\ddot{v}_r=-k_h g\ \sin\alpha+\ddot{v}_r \tag{2b}$$

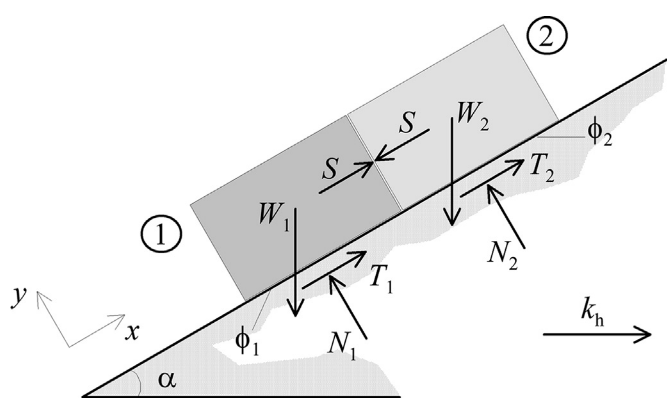

Fig. 1. Two rigid frictional blocks on inclined plane.

Assuming that the two blocks have the same relative displacement in the x direction ($u_{1r}=u_{2r}=u_r$) and thus $\ddot{u}_{1r}=\ddot{u}_{2r}=\ddot{u}_r$, and that no relative displacements occur in the y direction ($v_{1r}=v_{2r}=0$) and thus $\ddot{v}_{1r}=\ddot{v}_{2r}=0$, Eqs. (1) become

$$T_1-W_1(\sin\alpha+k_h\ \cos\alpha)-S=W_1\frac{\ddot{u}_r}{g} \tag{3a}$$

$$N_1-W_1(\cos\alpha-k_h\ \sin\alpha)=0 \tag{3b}$$

and

$$T_2-W_2(\sin\alpha+k_h\ \cos\alpha)+S=W_2\frac{\ddot{u}_r}{g} \tag{4a}$$

$$N_2-W_2(\cos\alpha-k_h\ \sin\alpha)=0 \tag{4b}$$

for block #1 and block #2, respectively. Incidentally, note that the condition $v_r=0$ implies that the normal forces at the base of the blocks are always positive, which, according to Eqs. (3b) and (4b), requires that

$$k_h<\frac{1}{\tan\alpha} \tag{5}$$

In the following, it is assumed that, even in static conditions, block #2 is in a limit state, *i.e.* that $T_2=T_{lim,2}=N_2\tan\phi_2$. This assumption, which is necessary to have a unique solution for Eqs. (3) and (4), holds if two conditions are satisfied, namely if $\tan\phi_2\le\tan\alpha$, and $\tan\ \phi_1>\tan\alpha+(W_2/W_1)(\tan\alpha-\tan\phi_2)$.

To understand the dynamic behaviour of the system, three conditions have to be examined: (1) the blocks and the base experience the same acceleration $k_h<k_c$; (2) a limit state (critical) condition is attained in the system, $k_h=k_c$; and (3) the two blocks start sliding along the base for $k_h>k_c$.

Until $k_h<k_c$ ($\ddot{u}_r=0$, $T_1<T_{\lim 1}$), no relative displacements occur between the blocks and the base, and the internal force between the two blocks is given by

$$S=W_2(\sin\alpha-\cos\alpha\ \tan\phi_2)+k_h W_2(\cos\alpha+\sin\alpha\ \tan\phi_2) \tag{6}$$

which is the sum of a static term proportional to the weight of block #2 and a dynamic term proportional to the inertia forces acting on the same block.

The critical condition for the system ($k_h=k_c$ and $\ddot{u}_r=0$) is attained when all the available friction is mobilized at the base of block #1, that is when $T_1=T_{\lim 1}$. From Eqs. (3) and (4) it is

$$k_c=\frac{(\tan\phi_1-\tan\alpha)+(W_2/W_1)(\tan\phi_2-\tan\ \alpha)}{(1+\tan\phi_1\ \tan\alpha)+(W_2/W_1)(1+\tan\phi_2\ \tan\alpha)} \tag{7}$$

Thus, the critical acceleration depends on both the mechanical and geometrical properties of the system ($\phi_{1,2}$ and α) and on the ratio of the weights of the two blocks, while the internal force is given by Eq. (6) with $k_h=k_c$.

When the applied horizontal acceleration exceeds the critical value ($k_h>k_c$), the two blocks start to slide along the base. The dynamic equilibrium of the system is expressed by

$$-\ddot{u}_r=\eta(k_h-k_c)g \tag{8}$$

where

$$\eta=\cos\ \alpha\frac{(1+\tan\phi_1\ \tan\alpha)+(W_2/W_1)(1+\tan\phi_2\ \tan\alpha)}{1+(W_2/W_1)} \tag{9}$$

is a coefficient which depends on the mechanical and geometrical properties of the system and on the ratio of the weights of the two blocks. Eq. (8) can be integrated twice, over the time intervals in which the relative velocity, $\dot{u}_r$, is non zero, to compute the permanent relative displacements experienced by the two blocks during the dynamic stage. During the same time intervals, the